Foundations of Engineering Mechanics

Ranjan Ganguli

Finite Element Analysis of Rotating Beams

Physics Based Interpolation

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Preface

Rotating beams are important mathematical models for structures such as helicopter rotor blades, wind turbine rotor blades, propellers, turbine blades and robotic manipulators. Therefore, the modeling and analysis of rotating beams is an important practical problem. The natural frequencies of the rotating beam should be kept away from multiples of the rotor speed. Therefore, accurate frequency prediction is important. Control of rotating beams requires the development of low-order models. This motivates the development of efficient mathematical models. Since the rotating beam equation does not have a simple exact solution, approximate methods such as Rayleigh–Ritz, Galerkin, and the finite element methods are widely used for the vibration analysis of rotating beams. This book provides an introduction to the finite element for rotating beams. A background on the Rayleigh–Ritz and Galerkin method is also provided.

The first chapter gives a detailed introduction to the rotating beam equation and illustrates the Rayleigh–Ritz, Galerkin, and finite element methods for its solution. Several example problems are given to illustrate the methods. A MATLAB finite element code for the rotating beam is also provided. The following chapters give adaptations of the basis functions which can accelerate the convergence of rotating beam finite elements, thus allowing for efficient low-order models. Some of these basis functions are based on analogies between the piano string and rotating Euler–Bernoulli and between the violin string and rotating Timoshenko beams. A major theme here is to choose finite element interpolation functions which are closer to the problem physics.

This book should be useful to engineers, graduate students and researchers working on rotating beam problems. It is also useful for people in the area of computational mechanics and applied mathematics.

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Contents

About the Author

Prof. Ranjan Ganguli obtained his M.S. and Ph.D. in Aerospace Engineering from the University of Maryland, College Park, in 1991 and 1994, respectively, and his B.Tech degree in Aerospace Engineering from the Indian Institute of Technology, Kharagpur, in 1989.

Following his Ph.D., he worked at the Alfred Gessow Rotorcraft Center of the University of Maryland as Assistant Research Scientist until 1997 on projects on rotorcraft health monitoring and vibratory load validation for the Naval Surface Warfare Center and United Technology Research Center, respectively. He also worked at the GE Research Lab in Schenectady, New York, and at Pratt and Whitney, East Hartford, Connecticut, from 1997 to 2000. He joined the Aerospace Engineering department of the Indian Institute of Science, Bangalore, as Assistant Professor in July 2000. He was promoted to Associate Professor in 2005 and to Full Professor in 2009. He is currently the Satish Dhawan Chair Professor at the Indian Institute of Science, Bangalore. He has held visiting positions at TU Braunschweig, University of Ulm and Max Planck Institute of Metal Research, Stuttgart, in Germany; University Paul Sabatier and Institute of Mathematics, in Toulouse, France; Konkuk University in South Korea, the University of Michigan, Ann Arbor, in USA, and the Nanyang Technological University, Singapore.

Professor Ganguli's research interests are in helicopter aeromechanics, aeroelasticity, structural dynamics, composite and smart structures, design optimization, finite element methods and health monitoring. He has published 178 articles in refereed journals and over 100 conference papers. He was awarded the ASME Best Paper Award in 2001, the Golden Jubilee Award of the Aeronautical Society of India in 2002, the Alexander von Humboldt fellowship in 2007 and the Fulbright Senior Research fellowship in 2010. Professor Ganguli is a Fellow of the American Society of Mechanical Engineers, a Fellow of the Royal Aeronautical Society, UK, a Fellow of the Indian National Academy of Engineering, a Fellow of the Aeronautical Society of India, an Associate Fellow of the American Institute of Aeronautics and Astronautics, and a Senior Member of IEEE.

He has taught courses on flight and space mechanics, engineering optimization, helicopter dynamics, aircraft structures, structural mechanics, aeroelasticity, and navigation. He has supervised the thesis of 15 Ph.D. and 35 Master's degree students. He has written books "Engineering Optimization" and "Gas Turbine Diagnostics," both published by CRC Press, New York, and books titled "Structural Damage Detection using Genetic Fuzzy Systems" and "Smart Helicopter Rotors," published by Springer.

Chapter 1
Introduction

1.1 Introduction

Rotating beams are useful mathematical models for helicopter rotor blades, wind turbine blades, turbine blades, robotic manipulators and propeller blades. Therefore, the rotating beam equation is a topic of great practical interest. In this chapter, we derive the equation of motion of the rotating beam and present some approximate methods for its solution. Approximate methods are very useful for obtaining frequencies of rotating beams. The Galerkin method and Rayleigh–Ritz method can be used for this purpose.

1.1.1 Elastic Blade

A good model of a rotor blade is an elastic beam restrained at the root, as shown in Fig. 1.1. Typically, helicopter rotor blades are treated as long slender beams. Treating the blade as a slender beam is appropriate since cross-sectional dimensions are much smaller than the length. At first, we will consider only out-of-plane or flap bending. Also, we make the Bernoulli–Euler assumption: a plane section normal to the neutral axis (beam axis) remains plane after deformation. It is also assumed that shear deflection is negligible and rotary inertia is neglected. This implies that the effect of rotation of element is small compared to vertical displacement. In addition, structural damping is neglected. With these assumptions, we are ready to derive the elastic blade governing differential equation.

Consider a blade section as shown in Fig. 1.2. Here $f_z(r, t)$ is the vertical load per unit length and $w(r, t)$ is the vertical deflection at station r. Several forces act on this blade section: $m\ddot{w}dr$ is the inertia force, f_z dr is the external force, $f_H dr$ is the axial force parallel to r, T is the axial force (Positive for tension), S is the shear force at the cut section (Positive as shown), M is the bending moment (Positive when top fiber is under compression).

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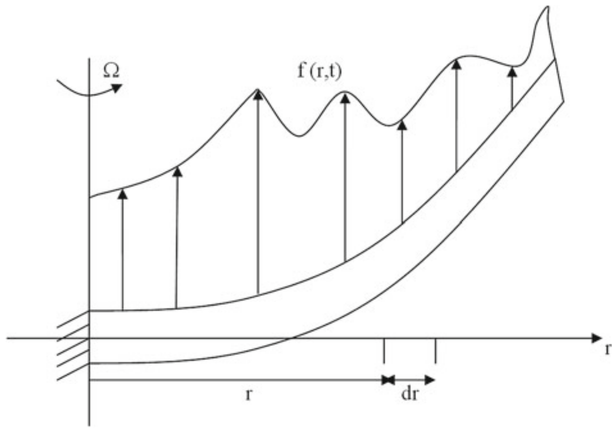
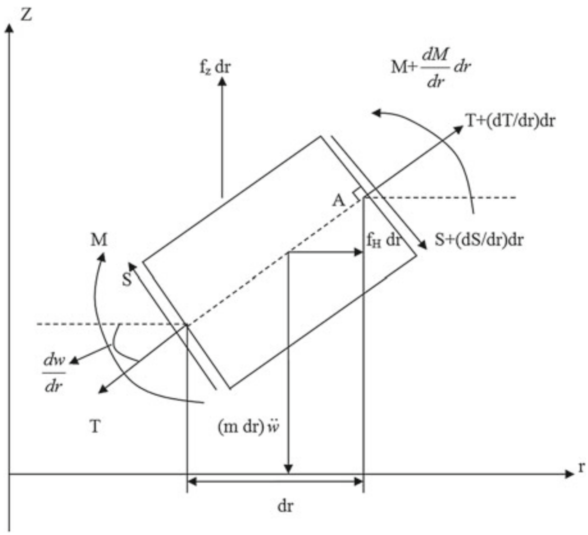


Fig. 1.1 Elastic blade with distributed load

Fig. 1.2 Elastic blade section



Also, $\frac{dw}{dr}$ is the bending deflection slope, assumed to be small. Therefore,

$$\cos\left(\frac{dw}{dr}\right) \approx 1 \qquad \sin\left(\frac{dw}{dr}\right) = \frac{dw}{dr} \tag{1.1}$$

Equilibrium of forces on element, in the r-direction yields

$$\sum F_r = -T + T + \frac{dT}{dr}dr + f_H dr - S\frac{dw}{dr} + \left(S + \frac{dS}{dr}dr\right)\left(\frac{dw}{dr} + \frac{d^2w}{dr^2}dr\right) = 0$$

$$\Rightarrow \frac{dT}{dr} + f_H + \frac{d}{dr}\left(S\frac{dw}{dr}\right) = 0 \tag{1.2}$$

Forces in the Z-direction yield,

$$\sum F_Z = f_z dr + S - S - \frac{dS}{dr}dr - m\ddot{w}dr - T\frac{dw}{dr} + \left(T + \frac{dT}{dr}\right)\left(\frac{dw}{dr} + \frac{d^2w}{dr^2}dr\right) = 0$$

$$\Rightarrow f_z - \frac{dS}{dr} - m\ddot{w} + T\frac{d^2w}{dr^2} + \frac{dT}{dr}\frac{dw}{dr} = 0 \tag{1.3}$$

Or

$$\frac{dS}{dr} = f_z - m\ddot{w} + \frac{d}{dr}\left(T\frac{dw}{dr}\right)$$

The above equation tells that differential of shear is loading distribution.

Now, we take moment about the point A in Fig. 1.2,

$$\sum M = M + Sdr - M - \frac{dM}{dr}dr = 0 \tag{1.4}$$

$$\frac{dM}{dr} = S$$

Thus, differential of moment is shear distribution. From beam theory,

$$M = EI\frac{d^2w}{dr^2} \tag{1.5}$$

Also,

$$S = \frac{dM}{dr} = \frac{d}{dr}EI\frac{d^2w}{dr^2} \tag{1.6}$$

$$\frac{\partial^2}{\partial r^2}\left(EI\frac{\partial^2 w}{\partial r^2}\right) + m\frac{\partial^2 w}{\partial t^2} - \frac{\partial}{\partial r}\left(T\frac{\partial w}{\partial r}\right) = f_z(r,t) \tag{1.7}$$

The above equation is the PDE for beam bending, Force equilibrium in horizontal direction gives,

$$\frac{dT}{dr} + f_H + \frac{d}{dr}\left(S\frac{dw}{dr}\right) = 0 \tag{1.8}$$

$$\frac{d}{dr}\left(S\frac{dw}{dr}\right) \rightarrow Second \;\; Order \;\; term \approx 0$$

$$\frac{dT}{dr} + f_H = 0. \tag{1.9}$$

To get a physical feel of the problem, we consider a rectangular cross-section as shown in Fig. 1.3.

$$I = \frac{bh^3}{12}$$

m = mass/unit length and E = Young's Modulus. The unit of I is m^4, unit of m is Kg/m and the unit of E is GPa.

1.1.2 Horizontal Force Equilibrium

Consider a case with uniform mass and a small mass element from Fig. 1.4. From Eq. (1.9) we get,

$$f_H = m\Omega^2 r \tag{1.10}$$

$$\frac{dT}{dr} = -m\Omega^2 r \tag{1.11}$$

Integrating,

$$T = -m\Omega^2 \frac{r^2}{2} + C \tag{1.12}$$

At tip, $r = R, T = 0, \Rightarrow C = m\Omega^2 \frac{R^2}{2}$

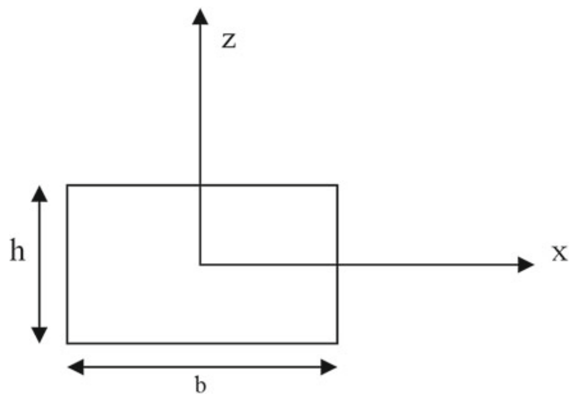


Fig. 1.3 Rectangular cross-section

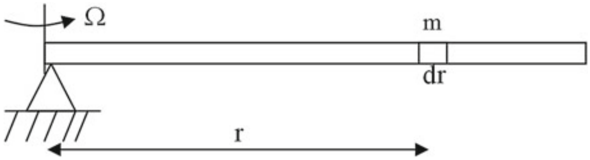


Fig. 1.4 Horizontal force equilibrium

$$T = \frac{m\Omega^2}{2}\left(R^2 - r^2\right) \tag{1.13}$$

In general,

$$T = \int_r^R m\Omega^2 r dr = \int_r^R m r \Omega^2 dr \tag{1.14}$$

1.1.3 Boundary Conditions

The governing equation is a fourth order partial differential equation (PDE) in space and needs 4 boundary conditions. These boundary conditions depend on the physics of the problem (Figs. 1.5, 1.6, 1.7, 1.8 and 1.9). We look at some typical boundary conditions.

(a) Pin end or simply supported (Fig. 1.5)

$$w = 0;\ M = EI\frac{d^2w}{dr^2} = 0$$

(b) Cantilevered or fixed end (Fig. 1.6)

$$w = 0;\ \frac{dw}{dr} = 0$$

(c) Free end (Fig. 1.7)

$$M = EI\frac{d^2w}{dr^2} = 0;\ S = \frac{d}{dr}\left(EI\frac{d^2w}{dr^2}\right) = 0$$

Fig. 1.5 Pinned end

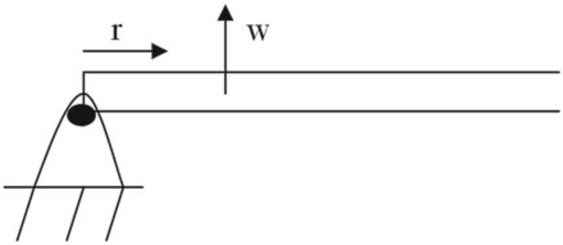


Fig. 1.6 Fixed end

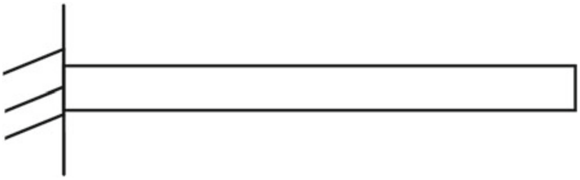
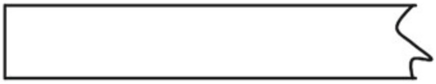


Fig. 1.7 Free end



(d) Vertical spring (Fig. 1.8)

$$M = EI\frac{d^2w}{dr^2} = 0;\ S = \frac{d}{dr}\left(EI\frac{d^2w}{dr^2}\right) = -Kw$$

(e) Leaf spring (Fig. 1.9)

$$M = EI\frac{d^2w}{dr^2} = K_\theta\frac{dw}{dr};\ S = \frac{d}{dr}\left(EI\frac{d^2w}{dr^2}\right) = 0$$

1.1.4 Initial Conditions

The beam PDE is second order in time and so needs two initial conditions: at $t = 0$, $\frac{\partial w}{\partial t}$ and w are prescribed.

1.1.5 Cantilever Beam Vibrations (Non-rotating)

Recall the PDE of the rotating beam,

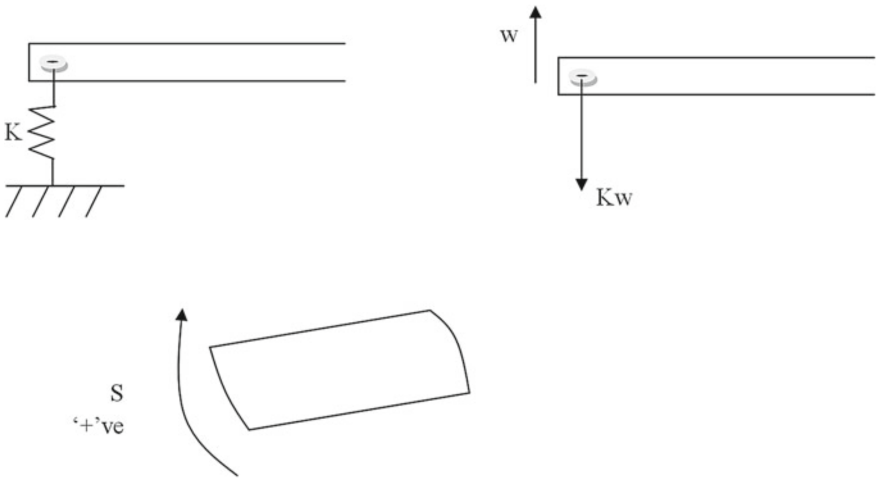


Fig. 1.8 Vertical spring

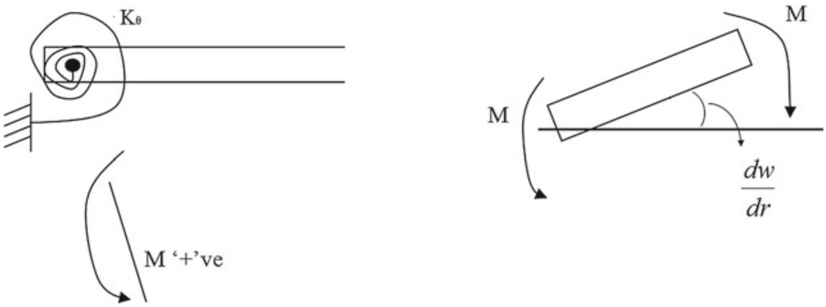


Fig. 1.9 Leaf spring

$$\frac{\partial^2}{\partial r^2}\left(EI\frac{\partial^2 w}{\partial r^2}\right) + m\frac{\partial^2 w}{\partial t^2} - \frac{\partial}{\partial r}\left(T\frac{\partial w}{\partial r}\right) = f_z(r,t) \tag{1.15}$$

For no rotation speed, $T = 0$. If we assume that the mass and stiffness are constant or uniform along the beam,

$$EIw'''' + m\ddot{w} = f_z(r,t) \tag{1.16}$$

If we consider the cantilever boundary conditions:

$$r = 0, w = 0, w' = 0$$
$$r = R, M = EIw'' = 0, w'' = 0; S = EIw''' = 0, w''' = 0$$