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ELEMENTS

OF THE

INTEGRAL CALCULUS,

WITH A

KEY TO THE SOLUTION OF DIFFERENTIAL EQUATIONS, AND A SHORT TABLE OF INTEGRALS.

BY

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SECOND EDITION, REVISED AND ENLARGED.

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PREFACE.

THE following volume is a sequel to my treatise on the Differential Calculus, and, like that, is written as a text-book. The last chapter, however, a Key to the Solution of Differential Equations, may prove of service to working mathematicians.

I have used freely the works of Bertrand, Benjamin Peirce, Todhunter, and Boole; and I am much indebted to Professor J. M. Peirce for criticisms and suggestions.

I refer constantly to my work on the Differential Calculus as Volume I.; and for the sake of convenience I have added Chapter V. of that book, which treats of Integration, as an appendix to the present volume.

W. E. BYERLY.

CAMBRIDGE, 1881.

PREFACE TO SECOND EDITION.

IN enlarging my Integral Calculus I have used freely Schlömilch's "Compendium der Höheren Analysis," Cayley's "Elliptic Functions," Meyer's "Bestimmte Integrale," Forsyth's "Differential Equations," and Williamson's "Integral Calculus."

The chapter on Theory of Functions was sketched out and in part written by Professor B. O. Peirce, to whom I am greatly indebted for numerous valuable suggestions touching other portions of the book, and who has kindly allowed me to have his Short Table of Integrals bound in with this volume.

W. E. BYERLY.

CAMBRIDGE, 1888.

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INTEGRAL CALCULUS.

CHAPTER I.

SYMBOLS OF OPERATION.

1. It is often convenient to regard a functional symbol as indicating *an operation to be performed upon the expression which is written after the symbol.* From this point of view the symbol is called a *symbol of operation*, and the expression written after the symbol is called the *subject* of the operation.

Thus the symbol D_x in $D_x(x^2y)$ indicates that the *operation* of differentiating with respect to x is to be performed upon the *subject* (x^2y).

2. If the *result* of one operation is taken as the *subject* of a second, there is formed what is called a *compound function.*

Thus $\log\sin x$ is a *compound function*, and we may speak of the taking of the $\log\sin$ as a *compound operation.*

3. When two operations are so related that the compound operation, in which the result of performing the first on any subject is taken as the subject of the second, leads to the same result as the compound operation, in which the result of performing the second on the same subject is taken as the subject of the first, the two operations are *commutative* or *relatively free.*

Or to formulate; if

$$fFu = Ffu,$$

the operations indicated by f and F are *commutative.*