



CliffsStudySolver™

# Trigonometry

By David Alan Herzog



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## About the Author

David Alan Herzog is the author of numerous books concerned with test preparation in mathematics and science. Additionally, he has authored over one hundred educational software programs. Prior to devoting his full energies to authoring educational books and software, he taught math education at Fairleigh Dickinson University and William Paterson College, was mathematics coordinator for New Jersey's Rockaway Township Public Schools, and taught in the New York City Public Schools.







# Dedication

This book is dedicated to Francesco, Sebastian, and Gino Nicholas Bubba, Rocio, Kira, Jakob and Myles Herzog, Hailee Foster, all of their parents, and Uncles Dylan and Ian.



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# Trigonometry Pretest

**Directions:** Questions 1 through 75.

Where it appears, the symbol  $\angle$  stands for “angle”;  $\angle$ s is its plural. You will need either a scientific calculator or the table of trigonometric functions on page 297 to answer certain questions. Where appropriate, approximate the value of  $\pi$  as 3.14.

Circle the letter of the appropriate answer.

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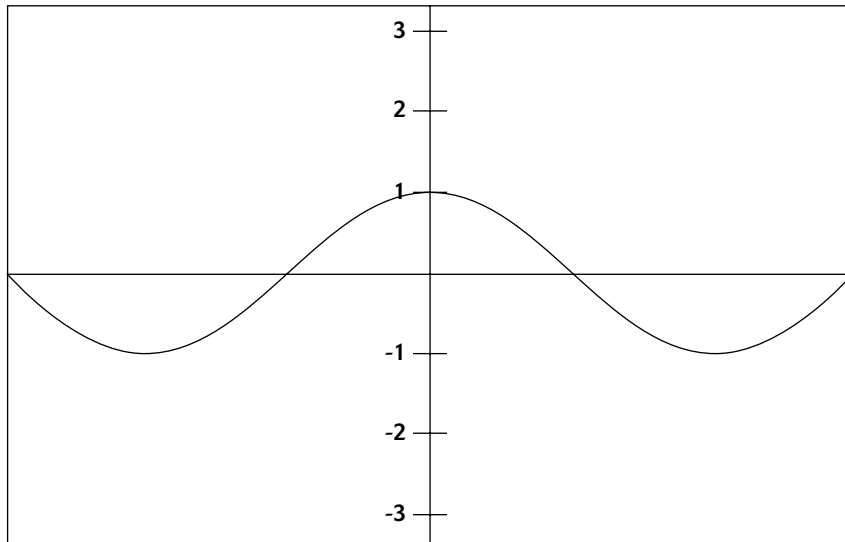
1. In which quadrant does a  $75^\circ$  angle in standard position have its terminal side?
  - A. I
  - B. II
  - C. III
  - D. IV
2. In which quadrant does a  $175^\circ$  angle in standard position have its terminal side?
  - A. I
  - B. II
  - C. III
  - D. IV
3. In which quadrant does a  $375^\circ$  angle in standard position have its terminal side?
  - A. I
  - B. II
  - C. III
  - D. IV
4. Which angle is coterminal with a  $45^\circ$  angle in standard position?
  - A.  $225^\circ$
  - B.  $295^\circ$
  - C.  $425^\circ$
  - D.  $765^\circ$

5. Which angle is coterminal with a  $125^\circ$  angle in standard position?
- A.  $205^\circ$
  - B.  $375^\circ$
  - C.  $485^\circ$
  - D.  $665^\circ$
6. Which of the following ratios gives the sine of an angle in standard position?
- A.  $\frac{\text{opposite}}{\text{adjacent}}$
  - B.  $\frac{\text{hypotenuse}}{\text{opposite}}$
  - C.  $\frac{\text{opposite}}{\text{hypotenuse}}$
  - D.  $\frac{\text{adjacent}}{\text{hypotenuse}}$
7. Which of the following ratios gives the tangent of an angle in standard position?
- A.  $\frac{\text{opposite}}{\text{adjacent}}$
  - B.  $\frac{\text{hypotenuse}}{\text{opposite}}$
  - C.  $\frac{\text{opposite}}{\text{hypotenuse}}$
  - D.  $\frac{\text{adjacent}}{\text{hypotenuse}}$
8. Which of the following ratios gives the cosecant of an angle in standard position?
- A.  $\frac{\text{opposite}}{\text{adjacent}}$
  - B.  $\frac{\text{hypotenuse}}{\text{opposite}}$
  - C.  $\frac{\text{opposite}}{\text{hypotenuse}}$
  - D.  $\frac{\text{adjacent}}{\text{hypotenuse}}$
9. In which quadrants is sine function negative?
- A. I and II
  - B. I and III
  - C. II and IV
  - D. III and IV



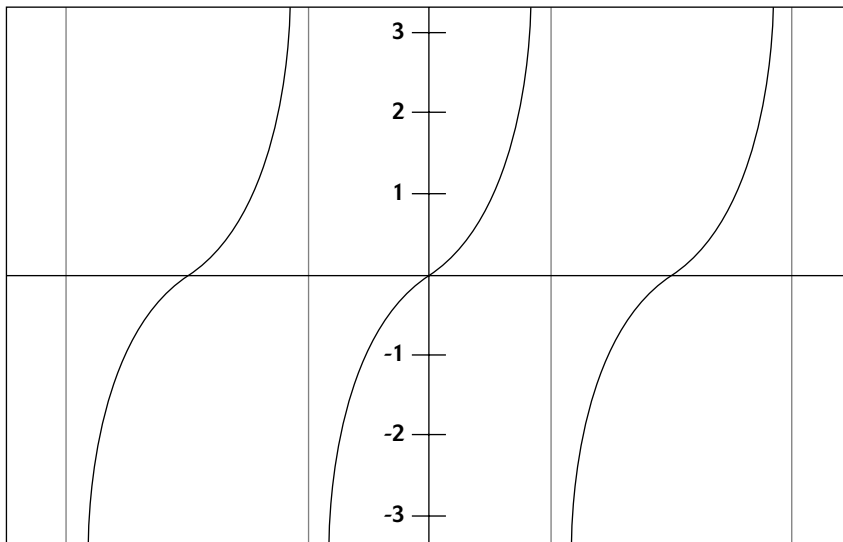
10. In which quadrants is tangent function negative?
- A. I and II
  - B. I and III
  - C. II and IV
  - D. III and IV
11. Given  $\tan 28.40^\circ = 0.5407$  and  $\tan 28.50 = 0.5430$ , what is  $\tan 28.43$ ?
- A. 0.5414
  - B. 0.5417
  - C. 0.5420
  - D. 0.5424
12.  $\cos 70^\circ = 0.3420$  and  $\cos 71^\circ = 0.3256$ . What is the cosine of  $70.6^\circ$ ?
- A. 0.3387
  - B. 0.3355
  - C. 0.3322
  - D. 0.3289
13. What is the degree measure of a circle's central angle that subtends an arc that is  $\frac{1}{3}$  the length of the circumference?
- A.  $90^\circ$
  - B.  $120^\circ$
  - C.  $150^\circ$
  - D.  $180^\circ$
14. What is the radian equivalent of the angle measure of a  $90^\circ$  angle?
- A.  $\frac{\pi}{4}$
  - B.  $\frac{\pi}{2}$
  - C.  $\pi$
  - D.  $2\pi$
15. What is the degree equivalent of the angle measure of  $\frac{2\pi}{3}$  radians?
- A.  $100^\circ$
  - B.  $110^\circ$
  - C.  $120^\circ$
  - D.  $130^\circ$

16. Given a unit circle with a right triangle drawn in standard position, the angle at the origin named  $\theta$ , and the hypotenuse being the radius of the circle, which of the following names the coordinates of the terminal side of the central angle?
- A.  $(\cos\theta, \tan\theta)$
  - B.  $(\sin\theta, \cos\theta)$
  - C.  $(\tan\theta, \sin\theta)$
  - D.  $(\cos\theta, \sin\theta)$
17. What is the domain of the sine function?
- A. the set of real numbers
  - B. the set of positive numbers
  - C. the set of negative numbers
  - D. all numbers from negative one to positive one
18. What is the range of the cosine function?
- A. the set of real numbers
  - B. the set of positive numbers
  - C. the set of negative numbers
  - D. all numbers from negative one to positive one
19. What is the period of an  $850^\circ$  angle?
- A.  $90^\circ$
  - B.  $110^\circ$
  - C.  $130^\circ$
  - D.  $150^\circ$
20. What is the period of an angle of  $7\pi$  radians?
- A.  $\frac{\pi}{2}$
  - B.  $\pi$
  - C.  $\frac{3\pi}{2}$
  - D.  $2\pi$



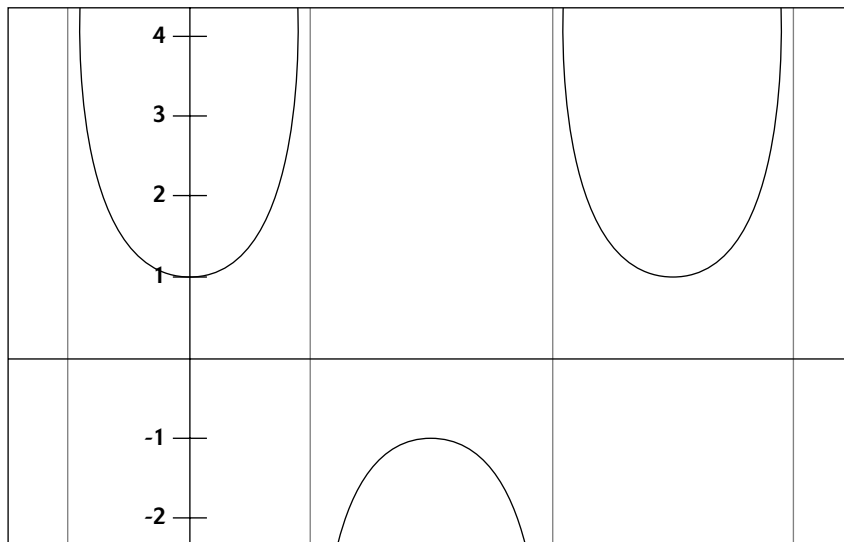
21. Which trigonometric function does the graph illustrate?

- A.  $y = \sin x$
- B.  $y = \cos x$
- C.  $y = \tan x$
- D.  $y = \sec x$



22. Which trigonometric function does the graph illustrate?

- A.  $y = \sin x$
- B.  $y = \cos x$
- C.  $y = \tan x$
- D.  $y = \sec x$



23. Which trigonometric function does the graph illustrate?
- A.  $y = \sin x$
  - B.  $y = \cos x$
  - C.  $y = \tan x$
  - D.  $y = \sec x$

For the following two problems, picture a right triangle,  $ABC$ , with sides  $a$ ,  $b$ , and  $c$  opposite  $\angle A$ ,  $\angle B$ , and  $\angle C$ , respectively.  $\angle C$  is the right angle.

24. If side  $c$  is 24 mm long and  $\angle A = 30^\circ$ , what is the length of side  $a$ ?
- A. 12 mm
  - B. 14 mm
  - C. 16 mm
  - D. 18 mm
25. If side  $c$  is 18 mm long and  $\angle B = 30^\circ$ , what is the length of side  $a$ ?
- A. 9.2 mm
  - B. 11.4 mm
  - C. 13.5 mm
  - D. 15.6 mm